Combination, permutation, and arrangement formulas

What are the formulas for calculating combinations, permutations, and arrangements? This article will guide you on how to calculate combinations and other related formulas.

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Permutations and combinations are among the most fundamental concepts in mathematics, relating to the selection of items from a group or set.

1. Permutations are the arrangement of items in a specific order from a given group.
2. A combination is the selection of items without considering the order.

Combination formula

Given a set A with n elements and an integer k (1 ? k ? n), each subset of A with k elements is called a combination of k elements from the n elements of A.

Formula for combinations of k elements from n

Picture 1 of Combination, permutation, and arrangement formulas

Formulas for the properties of combinations:

Picture 2 of Combination, permutation, and arrangement formulas

Picture 3 of Combination, permutation, and arrangement formulas

Picture 4 of Combination, permutation, and arrangement formulas

Examples of combinatorialism

Example 1:

A group consists of 12 students. How many ways are there to form a group?

a) Choose two students to represent the group.

b) Select two people, then assign them the roles of team leader and deputy team leader.

c) Divide the group into two subgroups, with the subgroup leader and deputy leader in different subgroups.

Solution

a) Choosing 2 friends from 12 friends is a combination of 2 from 12: C122 = 66 ways.

b) Choose 2 students and assign them roles using permutations of 2 from 12: A122 = 132 ways.

c) Divide the group into 2 teams, meaning each team has 6 students.

In which the team leader and deputy team leader are from different groups.

Choose 5 students from the remaining 10 to be in the same group as the group leader: C105 = 252 ways.

Choose 5 students from the remaining 5 to be in the same group as the deputy leader: C55 = 1 way.

So there are 252 x 1 = 252 ways.

Permutation formula

Let A be a set with n elements and let k be an integer (1 ? k ? n). When we select k elements from A and arrange them in a certain order, we obtain a permutation of k elements from n elements of A (referred to as a permutation of n elements from k of A).

The number of permutations of k elements from a set of n elements is:

Permutation formula:

Picture 5 of Combination, permutation, and arrangement formulas

1. Some conventions: 0! = 1, $A_n^0 = 1$, $A_n^n = n!$
2. Characteristics: This is an ordered sort, and the number of elements to be sorted is k: 0 ? k ? n.

For example:

Using the digits from 0 to 9, how many ways are there to form a natural number such that:

a) A number with 6 different digits

b) A six-digit number with different digits that is divisible by 10

c) An odd number with 6 different digits.

Solution

a) Form a six-digit number with different digits.

Choose the first digit from the numbers 1 to 9: there are 9 ways to choose.

The remaining digits are permutations of 5 of the remaining 9 numbers (other than the first digit) to form A95.

So there are 9A95 = 136080 numbers.

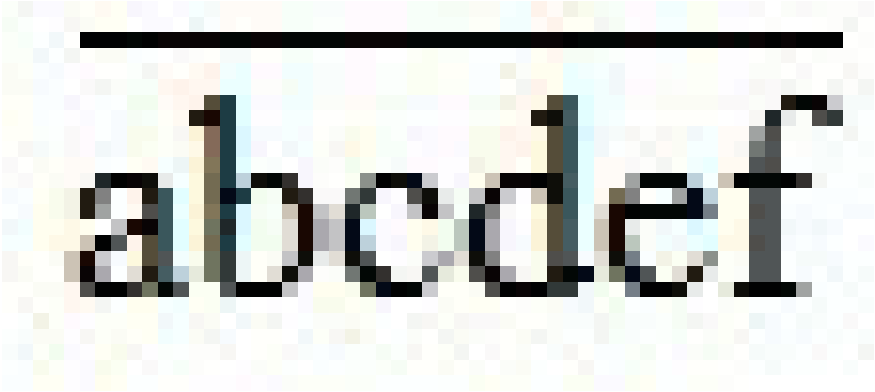
b) A six-digit number with different digits that is divisible by 10

Choose the digit for the units place: there is one way to choose it, which is the digit 0.

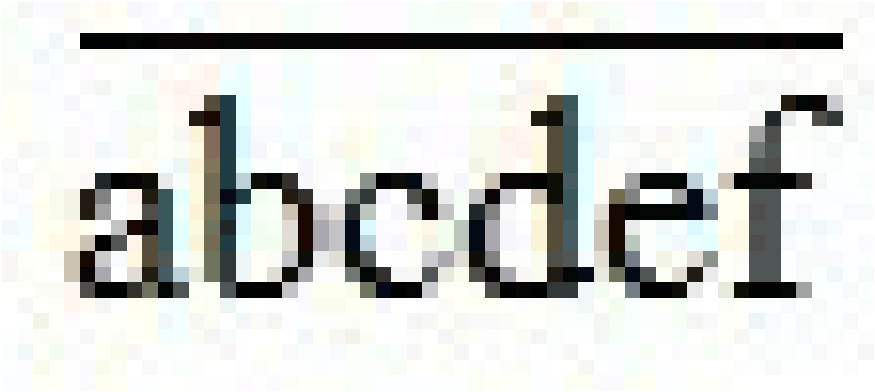
Choose the remaining digits to form a permutation of 5 of the remaining 9 numbers (excluding 0) such that A95

So there are A95 = 15120 numbers.

c) Let the number



be a 6-digit odd number with distinct digits formed from the digits 0 to 9.
Since



it is an odd number, f ?{1; 3; 5; 7; 9}
Choose f: there are 5 ways to choose

Choose a from the digits {1; 2; 3; 4; 5; 6; 7; 8; 9}{f}: there are 8 ways to choose

Choose b, c, d, e as permutations of 4 from the remaining 8 digits (different from f and a): we have A84

So there are 5.8A84 = 67200 numbers.

Permutations

a) Definition:

- Let A be a set of n elements (n ? 1).

Each result of arranging the n elements of set A in order is called a permutation of n elements.

Note: Two permutations of n elements differ only in their arrangement order.

b) Number of permutations:

- The symbol P_n denotes the number of permutations of n elements.

Permutation formula:

$P_n = n(n – 1)…2.1 = n!$

Convention: 0! = 1; 1! = 1.

Example: Arrange 10 students, including 5 boys and 5 girls, on a bench. How many ways are there to arrange them so that:

a) Arrange randomly

b) The boys are sitting next to each other.

c) Boys and girls sit alternately.

Solution

a) The number of ways to arrange 10 students on a bench is a permutation of 10: 10!

b) Arrange the boys to sit next to each other. We group 5 boys into a 'bundle': there are 5! ways to arrange them inside the 'bundle'.

Then, arranging 5 girls in one 'bundle' on a long bench can be done in 6! ways.

So there are 5! . 6! = 86400 ways to arrange the boys so they sit next to each other.

c) Suppose you arrange 10 students on a bench numbered from 1 to 10.

To arrange the boys and girls alternately.

Case 1: The boys sit in odd-numbered positions, and the girls sit in even-numbered positions.

Number of ways to arrange the boys: 5!

Number of ways to arrange the girls: 5!

Therefore, there are 5! . 5! ways to arrange them.

Case 2: The boys sit in even-numbered positions, and the girls sit in odd-numbered positions.

Similarly to the case above, we have 5! . 5! ways to arrange them.

So there are 2 . 5! . 5! = 28800 ways to arrange them.

The difference between permutations and combinations

The difference between permutations and combinations can be understood through the following table:

Permutations	**Combination**
In permutations, the order of arrangement is very important. For example, AB and BA are different combinations.	In a combination, the order of arrangement is not important. For example, AB and BA are the same combination.
A permutation is used when it is necessary to sort or classify different types of matter.	Combinations are used when you need to sort by type.
Permutation of two of the three given items a, b, c are ab, ba, bc, cb, ac, ca.	A combination is a combination of two things from three given things. a, b, c are ab, bc, ca.

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